THC Financial Engineering

THC White Paper Series #4
Low Interest Rate Regime: Challenges and Solutions

RE-EXAMINE INTEREST RATE MODELING

BONDS, LOANS, AND DERIVATIVE PRICING

by
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PROLOGUE

Interest rate models are central to valuing the embedded fixed-income options, which are prevalent in balance sheets, and are essential to enterprise risk management and market-making. They are also the bedrock to building credit and liquidity models, as well as income/profitability simulations. Financial economists cannot overstate the importance of interest rate models to our financial system.

The current low-interest-rate regime challenges the robustness of many operational interest rate models. The purpose of this paper is to explain the limitations of many operational interest rate models.

I will also show that by accepting the possibility of negative interest rates interest rate models can forecast interest rates as implied from the capital market.

INTRODUCTION

The current low-interest-rate regime has shaken financial economists to rethink many of the basic practices in the capital markets.

Is the nominal interest rate the sum of the real fundamental and inflation rate (Fisher Equation)?

Can the shape of the yield curve predict interest rates (Expectation Hypothesis)?

Are operational interest rate models valid when basic economic assumptions are in question?

Some media reports echo these questions: Kochkodin, Bloomberg News, September 2019, reported that “negative interest rates broke the Black Scholes model, Pillar of Modern Finance.” Gunjan Banerji, 10/17/2019, Wall Street Journal reported “Negative U.S Interest Rates? Option Traders Say Yes.” The urgency to re-evaluate interest rate models is critically apparent.

LOGNORMAL FLOOR RATE LIMITATION

In 1990s, financial economists categorically rejected the possibility of negative interest rates. The financial modelers focused on Lognormal Model. The model assumes that the change is rate is directly proportional to the rate level. And therefore when rate is low, the change becomes small and can never become negative.

The proportionality is called CEV, constant elasticity volatility. For now, I assume the lognormal model has CEV equaling one, directly proportional

The Lognormal Model assumes interest rates take random walks, analogous to that of the stocks, which always remain positive. The Lognormal Model projects rates rising rapidly, an undesirable attribute. Nearly 35 years of research sought to resolve these inconsistencies with observed market prices.

Figures 1 illustrate the complexity of interest rate simulations using a Lognormal Model. The figures depict the monthly one-month swap rates simulated over 30 years. The Lognormal Model generates 257 interest rate paths.

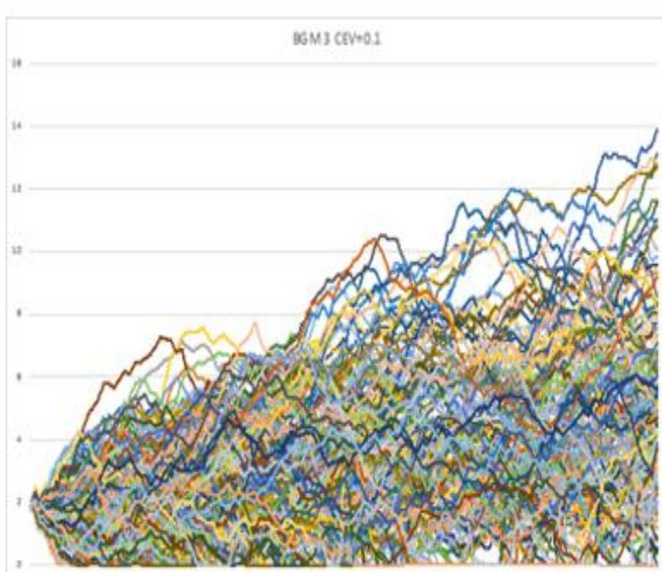

Figure 1 underscores the limitation of lognormal interest rate models.

LOGNORMAL FLOOR RATE LIMITATION

These interest rate paths are consistent with market pricing of the swaps and a portfolio of swaptions, satisfying the arbitrage-free model assumption, as explained in THC White Paper [1].

The simulation shows that the Lognormal Model does not allow negative interest rates, resulting in a positively skewed rate distribution.

Interest Rate Models should accept negative interest rates implied by swaption prices. The following section will provide empirical evidence of negative interest rates implied by the option prices.

LIMITATION ON SKEWNESS

Skewness is dynamic and has to be calibrated along with the term structure of volatilities

Lognormal Models use the Constant Elasticity Variance (CEV) to adjust the skewness of rate distributions with low CEV dialing down the skewness of the distribution. When CEV becomes small, the change in rate is less sensitive to the rate level. When CEV equals 0, then the change in rate is independent of the rate level.

In this example, Figure 1, the model CEV is 0.1, lowering the positive skewness by 90%. As a result, the simulated rates rise to approximately 10%.

Comparing Figure 1 and Figure 3, for most months, the tail of the rising rate distribution is consistent between the Local Volatility Model and the Lognormal CEV 0.1 Model. The rates are roughly capped at 10%, though not in September and November. For the tail of the falling rate distribution, the Local Volatility Model shows a positive value for zero strike floor derivatives, consistent with the Wall Street Journal report in October, as cited in this paper.

The Out-of-the-Money (OTM) determines the skewness of the rate distribution, as depicted in the September 2019 Rate Distribution Chart, showing that the OTM options are essential to specifying interest rate models, though many operating interest rate models use only the at-the-money options.

LIMITATION ON SKEWNESS

Figure 3. The time sequence of the Rate Distribution Charts from 3/2019 to 11/2019 shows the dynamic nature of the interest rate movements as perceived by the financial markets.

The results show that the Rate Distribution is dynamic, continually adjusting to the market perception of interest rate uncertainties and the skewness of the distribution. Many operating interest rate models estimate the skewness and floor rate separated from the model estimation process, the calibration, and the estimation is based on at-the-money options only, excluding out-of-the-money options. The next section will discuss the limitations in fixed-income pricing when the extent of projected negative rates and rate distribution skewness not dynamically estimated.

Calibration is explained in the White Paper 2019 "Embedded Option Pricing".

LIMITATION OF SAMPLING THE SCENARIO SPACE

The following paper will show that Monte-Carlo Simulations cannot have a sufficiently large sample to determine the distribution of interest rates inferred from the capital market prices. For this reason, the Monte-Carlo approach cannot provide interest rate forecast under normal and stressed scenarios.

Lognormal model rate distributions must be positive contrary to market experience of rate distribution

A market scenario can best explain the importance of using Rate Distribution. Consider Figure 3. On January 30, 2014, the yield curve was steep, with the 2-year forward curve. The one-month and 10-year rates were 1.26% and 3.57%, respectively. However, the capital market did not have to believe the one-month rate would rise from 0.19% to 1.26%, or the 10 year-rate from 2.98% to 3.57%, in 2 years. Instead, the capital market traded the caps to reflect the market rate expectation, creating a negative skewness in the rate distribution, while the Federal Reserve Bank affected a steep yield curve for a broader economic purpose.

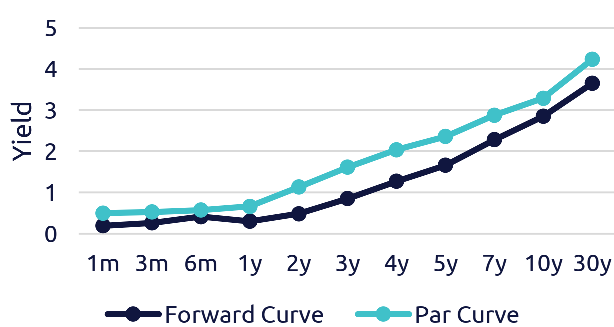

Figure 2. Yield Curves

Figure 3 depicts the swap curve and the 5-year forward curve on January 30,2014. As the capital market continued to recover from the 2008 financial crisis, the Federal Reserve Bank held the one month rate low at 19pb, but the capital market anticipates a robust recovery trading the 10-year rate at 2.97%.

Swap as of 01/30/2014

Time	1m	3m	6m	1y	2y	3y	4y	5y	7y	10y	30y
Par Curve	0.190	0.260	0.410	0.300	0.480	0.850	1.270	1.660	2.280	2.850	3.650
Spot Curve	0.190	0.260	0.410	0.300	0.480	0.854	1.283	1.687	2.346	2.977	4.030
Forward Curve	0.495	0.525	0.571	0.661	1.131	1.611	2.035	2.358	2.875	3.287	4.232

IMPLICATIONS ON OPTION PRICING

Distribution of one-month rates can be negative, distribution skewness dynamic

Provided is a historical trend of the Probability Distributions of Rates. Rate Distribution depict the projected one-month interest rates specified by the Local Volatilities Model.

I chose the period 3/2019 to 11/2019 to depict the period Rate Distribution as implied by the At-the-Money and Out-of-the-Money swaptions. Presented is the changing market views, as inferred by in-the-money and out-of-the-money swaption prices.

The results show that some projected implied rates have been significantly negative since August 2019

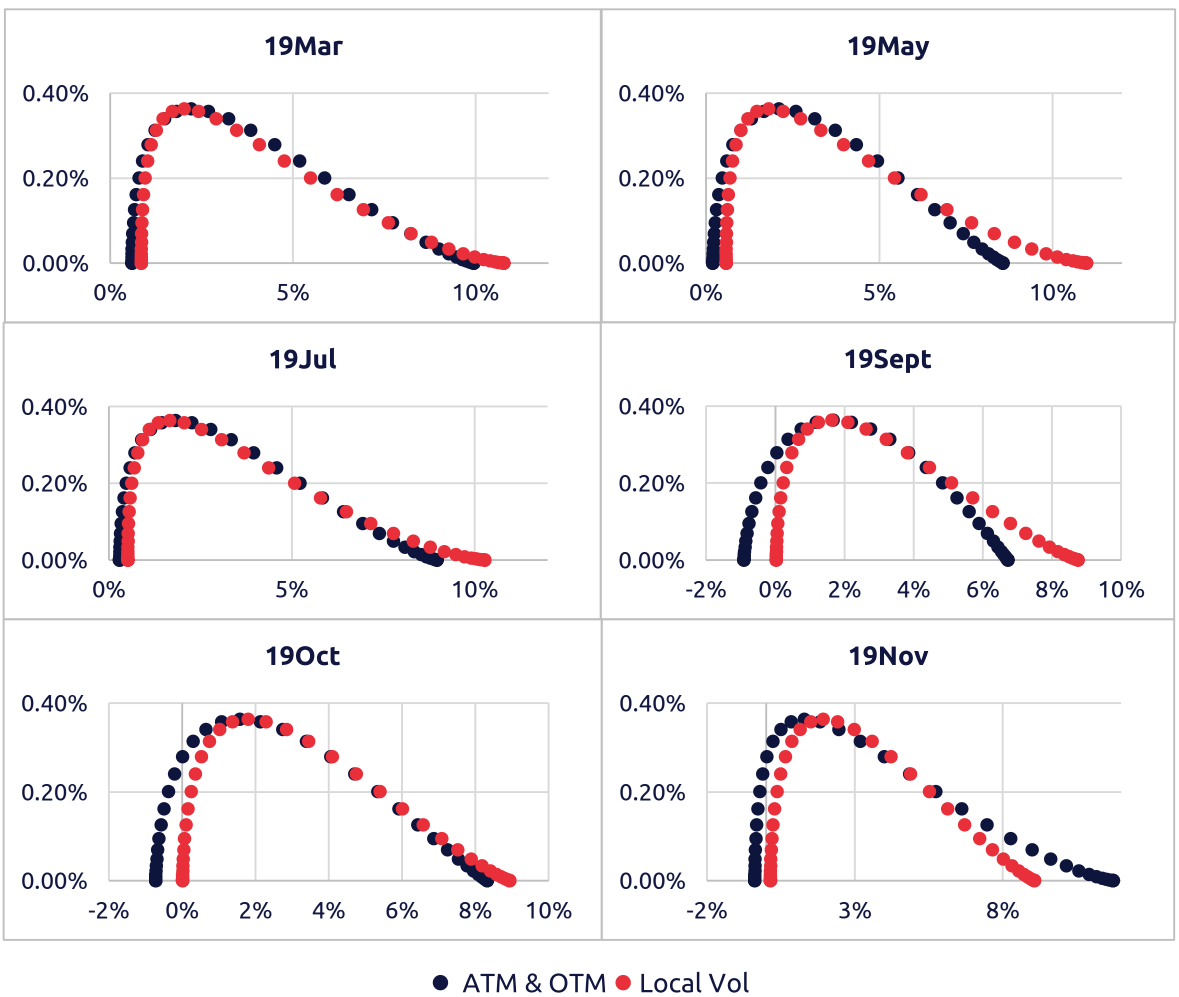

Figure 3. ATM & OTM Vol vs Local Vol

CONCLUSIONS

Perils of using legacy models developed in the '90.

Many interest rate models do not dynamically accept

- *The market implied level of negative interest rates*
- *Do not measure skewness using OTM*
- *Cannot cover many interest rate scenarios*

Financial economists have categorically rejected the possibility of negative interest rates for over 35 years, resulting in much-unwarranted research and the development of interest rate models. Today, the low-interest-rate regime has "broke(n) the Black Scholes model, Pillar of Modern Finance." Economists should critically evaluate the current operational interest rate models.

This paper shows:

Distribution Skewness. The interest rate model should use both capital market pricing of At-the-Money and Out-of-the-Money options to define the Rate Distribution, the projected interest rate minimum and the distribution skewness.

The Expectation Hypothesis. The interest rate model's Rate Distribution determines the market expected interest rate level, which is a more appropriate rate forecast than that suggested by the Forward Curve, as the Expectation Hypothesis suggests.

The Fisher Equation. The concept that the nominal interest rate is the sum of the economy's real rate of return plus the inflation rate (the Fisher Equation) is a positive theory. By way of contrast, the arbitrage-free model is a normative theory. The arbitrage-free model can lead to timely and actionable decisions and does not rely on the concept of general market equilibrium.

EPILOGUE

I have introduced the Local Volatility Model in a series of papers.

My **White Paper #1** introduces the Local Volatility Model.

Paper #2 presents a profitability model for assets and liabilities that have embedded options.

Paper #3 uses historical data to show the prevalence of embedded options on the balance sheet and shows the market-implied rate distributions in the low-interest rate regime.

This paper continues the research. On one hand, this paper cautions the use of some of the current operational interest rate models. On the other hand, this paper suggests a new approach to infer market forecast of interest rates and their uncertainties. In the next paper,

I will describe the implementation issues of an interest rate model sampling just 257 random rate paths (in this paper example) from 2^{360} scenarios to price options and show the impact of using the Local Volatility model in option pricing as opposed to some of the current operational interest rate models.

I will then introduce Market Interest Rate Forecast (MIRF), an interest rate forecasting model based on capital market pricing on options.

TECHNICAL NOTES

Financial economists have made tremendous progress in interest rate modeling in the past 35 years. Models such as Ho-Lee, Black-Derman-Toy, Cox-Ross-Ingersoll, Hull-White, Black-Karasinski, Brace-Gatarek-Musiela, and Longstaff-Santa-Clara-Schwartz continually enhance interest rate modeling. These models have also introduced many new concepts such as martingale, lattice, recombining, delta hedge, risk-neutral measure q, physical measure p, kernel, vol surface, lognormal & normal models, OAS and Greeks, string theory, stratified sampling, rational option exercise rules, and CEV skew model. The study of interest rate modeling has even become a core course in the mathematics department.

For the following equations, I use the notations:

r = a short-term rate

$\emptyset$= adjustment factor in ensuring the interest rate movements are arbitrage-free

$\sigma(t)$ = term structure of volatilities

dz= wiener process; normal distribution over a short time

The basic models from which extend many other models for credit risks, yield curve movements, computational efficiencies:

$$dr = \emptyset(t)dt + \sigma(t)dz$$

The normal model where the term structure is independent of the rate level. The Heath-Jarrow-Morton (HJM) models determine the term structure of volatilities using the forward rates.

$$dr = \emptyset(t)rdt + \sigma(t)rdz$$

The lognormal model, where the term structure of volatilities measures the proportional change of rates. Black-Derman-Toy (BDT) avoids negative rates.

$$dr = \emptyset(t)(\, l - r)dt + \sigma(t)r^{0.5}dz$$

TECHNICAL NOTES

The Cox – Ross – Ingersoll (CIR) model where the rates mean revert to some long-term rate and rate change depends on the term structure of volatilities and positively related to the rate level

$$dr = \emptyset(t,r)dt + \sigma(t)r^{\alpha}dz$$

Constant Elasticity Variance model where elasticity α can be between 1 and 0.1. BGM (LIBOR Market Model) is a CEV model with an efficient algorithm that fits precisely to market pricing of swaptions. These models do not accept negative rates. To allow for negative rates, BGM and Hull-White models allow for a modeler to set the floor rate, which can be negative.

$$dr = \emptyset(t)dt + \sigma(t)\eta(r)dz$$

Local Volatility Model allows for the out-of-money options to specify the rate distribution enabling the rate distribution $\eta(r)$ to fit the out-of-the-money option prices.

The established interest rate models do not allow for the swaption prices to specify the rate distribution. Instead, these models impose the swaptions be priced based on a form of lognormal models or a normal model with a constraint of the minimum rate. The Local Volatility Model seeks to overcome these problems in a low-interest rate regime by explicitly determining the rate distribution $\eta(r)$ from the OTM option prices.

About the Author

Thomas S. Y. Ho. Ph.D., is CEO and Founder of Thomas Ho Company Ltd., a New York-based financial engineering company. He is the co-author of the Ho-Lee Model and has over 35 years of experience in financial modeling and is the author of more than 80 books and refereed publications.

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