



THC Financial Engineering
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Risk Modeling Bulletin Issue 3

Interest Rate Volatilities

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This issue deals with the level of interest rate risk, i.e., interest rate volatilities. In particular, we discuss the impact of interest rate volatilities on any financial instrument as measured by the instrument's convexity. We also discuss the Black volatility surface of June 2006, showing how the market prices the volatilities.

Feature Article: Convexity and Interest Rate Volatilities

The proportional change in the price of a fixed-income financial instrument is not linearly related to the shift in the spot yield curve as specified by the effective duration, especially when the shift of the spot curve is large. How should we then adjust the duration model to incorporate large changes in interest rates?

The relevant measure is called convexity. The proportional change of the value of a financial instrument is the sum of the change due to duration and to convexity. More precisely,

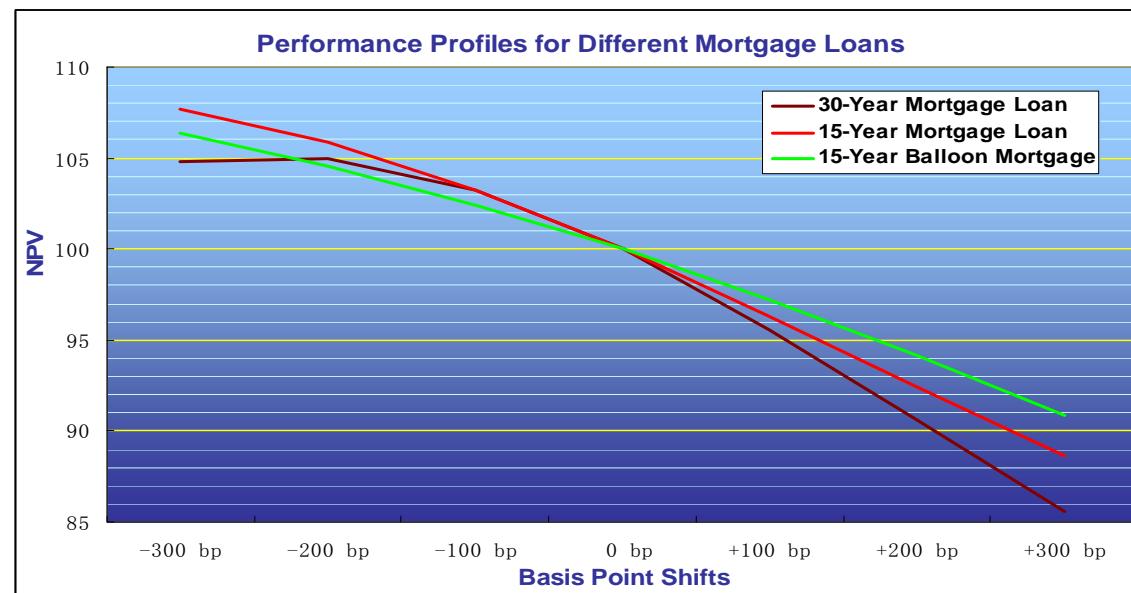
$$\frac{\Delta P}{P} = -\text{Duration} * \Delta r + \frac{1}{2} * \text{Convexity} * \Delta r^2.$$

The convexities for a sample of three types of fixed-rate mortgage loans generated from THC Decisions are reported below in Table 1. Figure 1 shows the mortgage performance profiles for the March 2005 cycle.

Table 1: Convexities of Different Mortgage Loans

	WAC(%)	Start Date	Maturity	OAS(%)	Convexity
30-Year Mortgage Loan	6.32	2005-3-31	2034-11-30	1.09	-133.59
15-Year Mortgage Loan	6.32	2005-3-31	2017-10-31	0.94	-43.39
15-Year Balloon Mortgage	6.32	2005-3-31	2011-1-31	1.43	-33.65

FIGURE 1



When interest rates fall, mortgagors will have an incentive to prepay. As such, mortgagors hold call options against the banks. As a result, mortgage loan values rise at a slower rate for decreases in rates when interest rates are low. For the 30-year mortgage loan, the proportional change in price as predicted by the duration model has to be

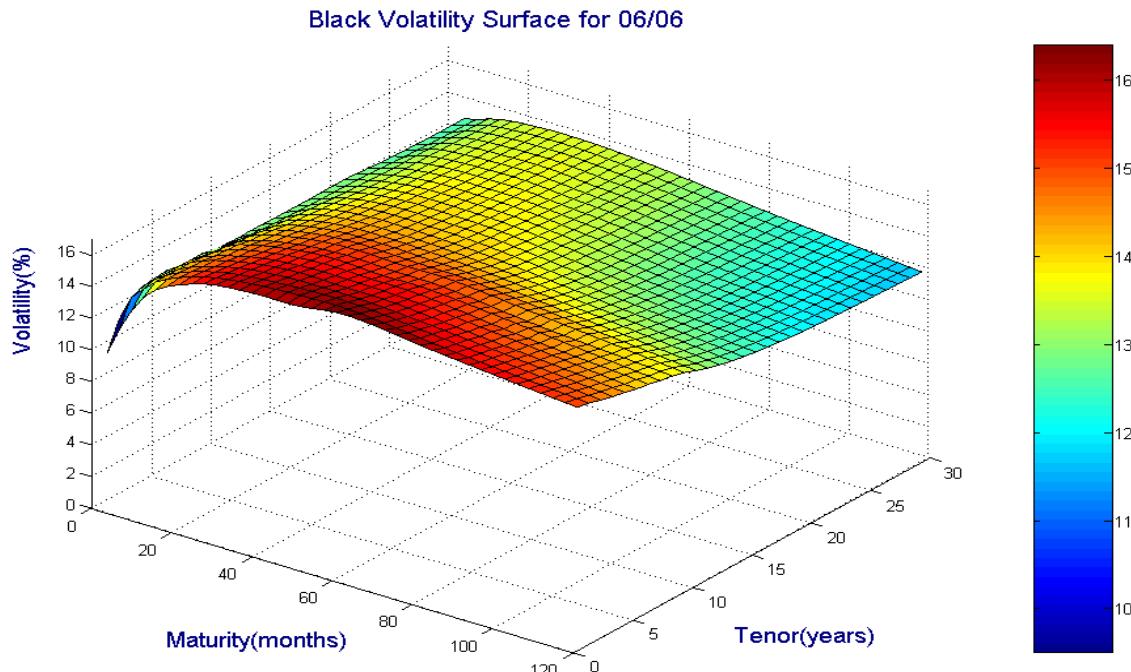
adjusted by a convexity factor of -133.59 . In other words, for a 100 basis point shift, the proportional change in value is adjusted downwards by 0.66795 percent ($= -0.5 \times 133.59 \times 0.0001$)

This so-called negative convexity explains the results in funding the mortgage loans with saving accounts in Bulletin 1. As you will recall, we showed that profit in a period of falling interest rates is capped by the negative convexity caused by prepayments. Negative convexity, however, is not all bad. Banks are compensated by the higher mortgage interest payments. Therefore, the convexity measure enables the banks to weigh the risk and return tradeoff, given their view of the market.

Reference: Convexity is discussed in Ho and Lee 2001 [Oxford Guide to Financial Modeling](#), Oxford University Press pp 69

Market Perspective: Black Volatility Surface for 06/06

FIGURE 2



How are interest rate volatilities measured? Banks can, of course, measure the standard deviations of historical interest rates. But history is not a good predictor of the future. Because of this, the embedded options found in the balance sheet, like the call options in mortgages, cannot be valued using historical volatilities. A better approach is to use the prices of swaptions traded in the capital markets to infer the volatilities used by traders. These volatilities are called implied volatilities. The market's standard measure is based on the Black formula, and they are called Black volatilities. These are calculated from the prices of swaptions for a given maturity (the time to the expiration of the option) and tenor (the maturity of the underlying of the fixed rate swap of the option).

Figure 2, for June 2006, shows that the implied volatilities tend to decline as the maturity and tenor increase. The implied volatilities peak at around the 30-month maturity. Sometimes, the market perceives a higher or lower risk in the 30-month range.

Reference: volatility surface and the implied volatility functions are discussed in Ho and Lee 2001 [Oxford Guide to Financial Modeling](#), Oxford University Press pp 179-192.

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Contact us if you have any questions, suggestions, or comments

support@thomasho.com Voice: 1-212-732-2878 Fax: 1-212-608-1233
[Http://www.thomasho.com](http://www.thomasho.com) 55 Liberty Street, 4B, New York, NY 10005-1003 USA

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